

Simultaneous Generation of WIMP Miracle-like Densities of Baryons and Dark Matter

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The observed density of dark matter is of the magnitude expected for a thermal relic weakly-interacting massive particle (WIMP). In addition, the observed baryon density is within an order of magnitude of the dark matter density. This suggests that the baryon density is physically related to a typical thermal relic WIMP dark matter density. We present a model which simultaneously generates thermal relic WIMP-like densities for both baryons and dark matter by modifying a large initial baryon asymmetry. Dark matter is due to $O(100)$ GeV gauge singlet scalars produced in the annihilation of the $O(\text{TeV})$ coloured scalars which are responsible for the final thermal WIMP-like baryon asymmetry. The requirement of no baryon washout implies that there are two gauge singlet scalars. The low temperature transfer of the asymmetry to conventional baryons can be understood if the long-lived $O(\text{TeV})$ coloured scalars have large hypercharge, $|Y| > 4/3$. Production of such scalars at the LHC would be a clear signature of the model.

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I. INTRODUCTION

The observed density of baryons and of dark matter are within an order of magnitude of each other. If we discount simple coincidence as an explanation, there are broadly two approaches to understanding the baryon to dark matter ratio. One is the simultaneous production of baryons and dark matter, usually via decay of a particle to similar number of baryons and dark matter particles, as would be expected if there was a conserved charge carried by both [1–5]. (This is closely related to models of asymmetric dark matter, which have been a focus of recent interest [6].) In this case we expect $n_{DM} \sim n_B$ and so $m_{DM} \sim m_n = 1 \text{ GeV}$. (Models exist which break this simple relation [7–13].) The other is anthropic selection. An example of this is the case of axion dark matter, where superhorizon domains with different dark matter densities can be generated and $\rho_{DM} \sim \rho_B$ may then be anthropically favoured by the baryon density in galaxies [14].

However, these approaches generally neglect the other notable coincidence of the dark matter density, its similarity to the thermal relic density of particles whose mass and interactions are characterized by the weak scale, the so-called "WIMP miracle". If the WIMP miracle is not a coincidence but instead an indication of the process responsible for dark matter, and if we discount anthropic selection, then the baryon asymmetry must also be related in some way to the WIMP miracle.

Thus there are three possibilities (excluding coincidence): (i) the WIMP miracle is the origin of the dark matter density and the baryon asymmetry is physically related to the WIMP miracle, (ii) the WIMP miracle is the origin of the dark matter density and the baryon asymmetry is related to this by anthropic selection, and (iii) the WIMP miracle is not the explanation of dark matter.

The question of whether (i) is possible is therefore fundamentally important. If such a mechanism exists, it would be possible to understand both of the coincidences of the dark matter and baryon densities, why they related to each other and to the WIMP miracle, in terms of particle physics. If not, it would tell us that either (ii) or (iii) is true i.e. either anthropic selection plays an essential role or the WIMP miracle is just a coincidence, not related to the origin of dark matter.

In [15] we proposed a model which could account for a thermal WIMP-like density of baryons by modifying a large initial baryon asymmetry via a weak-strength B-violating annihilation process, a process we call baryomorphosis. The baryon asymmetry is initially locked in a density of particles which are decoupled from the thermal Standard Model (SM) background [15]. These particles decay to pairs of scalar particles $\phi_B, \hat{\phi}_B$ of mass $O(1) \text{ TeV}$ ('annihilons'). ϕ_B and $\hat{\phi}_B$ have opposite gauge charges but, importantly, not opposite baryon number. They annihilate to final state scalars via a B-violating, naturally weak-strength interaction. If the temperature at which the baryon number is transferred to annihilons is less than the freeze-out temperature of the B-violating interaction, a non-thermal but thermal WIMP-like relic density of ϕ_B and $\hat{\phi}_B$ will remain. These subsequently decay to conventional baryons, leaving a baryon density which is naturally similar to a thermal relic WIMP dark matter density.

While the original model in [15] demonstrated that it might be possible to understand why the baryon asymmetry is similar to a thermal relic WIMP density, it also highlighted some obstacles to be overcome in the construction of a natural model. Since the question we wish to answer is whether there exists a plausibly natural extension of the Standard Model (SM) which can account for a thermal WIMP-like baryon asymmetry, the naturalness of its construction is an important issue.

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One issue is the danger of washout of the baryon asymmetry by the B-violating annihilation process. This excluded the possibility that ϕ_B and $\hat{\phi}_B$ could couple to the Higgs bilinear $H^\dagger H$, since this would induce a B-violating mixing of ϕ_B and $\hat{\phi}_B$ once the Higgs VEV was included, leading to baryon washout via scattering of ϕ_B and $\hat{\phi}_B$ from the thermal background [15]. Therefore ϕ_B and $\hat{\phi}_B$ must only couple to scalars which do not have a VEV. In addition, a tree-level $\phi_B \hat{\phi}_B$ mixing term must be excluded and loop corrections should not generate a dangerous mixing between ϕ_B and $\hat{\phi}_B$. We will show how such dangerous terms may be excluded via a simple discrete symmetry.

A second issue concerns the decay of ϕ_B and $\hat{\phi}_B$ to conventional baryons. This must occur after the $\phi_B \hat{\phi}_B$ density annihilates to its final form, which occurs at the decay temperature of the initial asymmetry T_d . Since typically $T_d \lesssim 100$ GeV when the B-violating process is due to TeV scale particles, the lifetime of ϕ_B and $\hat{\phi}_B$ must be long, $\gtrsim 10^{-10}$ s. We therefore need to understand why large renormalizable couplings between ϕ_B and $\hat{\phi}_B$ and conventional SM fermions, which would lead to rapid decay, are excluded.

In the original baryomorphosis model [15], the nature of the dark matter particle was not addressed; it was simply assumed to be a conventional WIMP. However, since we need to introduce new scalar particles to serve as the final state of the B-violating annihilation process and a new discrete symmetry to control B-washout, a natural possibility is that these new scalar particles could also account for dark matter which is stabilized by a discrete symmetry. In this case the decay of the large initial baryon asymmetry can lead to a final baryon asymmetry and a dark matter density which are both similar to a typical thermal relic WIMP density.

In this paper we will present a simple scalar extension of the SM which can account for thermal relic WIMP-like densities of both baryons and dark matter. The paper is organized as follows. In Section 2 we specify the model and its discrete symmetries. In Section 3 we discuss the modification of the large initial baryon asymmetry to a thermal WIMP-like baryon asymmetry and the production of a scalar dark matter density. In Section 4 we present the baryon asymmetry and dark matter density as a function of the masses and couplings of the scalars. In Section 5 we discuss the transfer of the baryon asymmetry to conventional baryons. In Section 6 we present our conclusions. In the Appendices we discuss the slowing of the annihilons and gauge singlet scalars by scattering from the thermal background and we provide the annihilation cross-section for the gauge singlet scalars.

II. THE MODEL

The model is based on a pair of scalars ('annihilons') ϕ_B and $\hat{\phi}_B$, with mass $O(\text{TeV})$ and with opposite gauge charge. (As we will discuss, although it is possible to construct a model with gauge singlet annihilons, the annihilons must carry either a global or gauge charge, strongly suggesting that they have SM gauge charges.) To be specific, we will focus on the case where the annihilons ϕ_B and $\hat{\phi}_B$ are colour triplets, transforming as $(\mathbf{3}, \mathbf{1})$ and $(\bar{\mathbf{3}}, \mathbf{1})$ under $SU(3)_c \times SU(2)_L$; other charge assignments will have a similar cosmology.

We first consider the symmetries that are required to evade $\phi_B \hat{\phi}_B$ mixing that could lead to baryon washout. In [15] the B-violating interaction allowing the annihilation of the annihilons was assumed to be of the form

$$\mathcal{L}_{\phi_B \hat{\phi}_B \text{ ann}} = \lambda_B \phi_B \hat{\phi}_B \hat{H}^\dagger \hat{H} + h.c. \quad (1)$$

Here \hat{H} is a scalar which develops no expectation value. However, such an interaction means that a mixing term of the form $\Delta m \phi_B \hat{\phi}_B$ cannot be excluded by any symmetry. There is also no symmetry which can exclude an interaction with the Higgs of the form $\lambda \phi_B \hat{\phi}_B H^\dagger H$, generating a mixing term with $\Delta m^2 = \lambda_B < H >^2$. Such mixing terms will generally lead to washout of the baryon asymmetry for natural values of the couplings, for example via scattering from the thermal background via gauge boson exchange, which imposes the constraint $\Delta m \gtrsim 0.1$ GeV [15]. This requires that $\lambda_B \lesssim 10^{-7}$. Such couplings are not consistent with a thermal WIMP-like baryon asymmetry from annihilation of TeV-scale particles via Eq. (1), which requires $\lambda_B \sim 0.1$.

Moreover, the interaction Eq. (1) will lead to a quadratic divergent term of the form $\Delta m \phi_B \hat{\phi}_B$. Quadratic divergent contributions are not necessarily a problem. The situation is similar to the case of the Higgs boson mass in the SM. This is treated as a phenomenological input and the quadratic divergence is absorbed into the physical mass by renormalization. The same can be true for Δm . However, such a solution is not acceptable if the theory is considered a low energy effective theory with a physical cutoff $\Lambda \gtrsim 1$ TeV. In this case the quadratic divergence would be considered a real contribution to Δm , requiring $\lambda_B \lesssim 10^{-6}$.

These problems can be solved if the product $\phi_B \hat{\phi}_B$ transforms under a discrete Z_2 symmetry such that $\phi_B \hat{\phi}_B \rightarrow -\phi_B \hat{\phi}_B$. This can be achieved by introducing a discrete symmetry Z_A and real scalars s and \hat{s} , where Z_A is defined by

$$\phi_B \rightarrow \phi_B; \quad \hat{\phi}_B \rightarrow -\hat{\phi}_B; \quad s \rightarrow s; \quad \hat{s} \rightarrow -\hat{s}, \quad (2)$$

with all SM fields invariant under Z_A . Z_A excludes terms of the form $\phi_B \hat{\phi}_B$ and $\phi_B \hat{\phi}_B H^\dagger H$ but allows the interaction term

$$\mathcal{L}_{\phi_B \hat{\phi}_B \text{ ann}} = \lambda_B \phi_B \hat{\phi}_B s \hat{s} + h.c. \quad (3)$$

It also excludes the dangerous interactions $\phi_B \hat{\phi}_B s s$ and $\phi_B \hat{\phi}_B \hat{s} \hat{s}$, which would generate quadratic divergent $\phi_B \hat{\phi}_B$ mixing terms.

Note that if ϕ_B and $\hat{\phi}_B$ were gauge singlets, the B-violating mass term $\phi_B \hat{\phi}_B$ would not be excluded, nor would the term $\phi_B \hat{\phi}_B s s$ which leads to a quadratic divergent mixing. To exclude these, ϕ_B and $\hat{\phi}_B$ must also be oppositely charged with respect to either a global or a gauge symmetry. It is therefore natural to assume that ϕ_B and $\hat{\phi}_B$ carry SM gauge charges.

The SM requires a dark matter candidate. In the context of the present model the simplest possibility is to consider one or both of s and \hat{s} to be dark matter particles. We will consider the simplest case where s and \hat{s} are real gauge singlet scalars. (For discussions of gauge singlet scalar dark matter see [16–20].) Models based on complex singlets or inert doublets [21] could also be constructed. To ensure that the gauge singlet scalars are stable dark matter particles, we introduce an additional discrete symmetry Z_S , under which s and \hat{s} are odd and all other particles are even. We then expect couplings to the SM of the form

$$\frac{\lambda_s}{2} s s H^\dagger H + \frac{\lambda_{\hat{s}}}{2} \hat{s} \hat{s} H^\dagger H. \quad (4)$$

These couplings will allow the s and \hat{s} densities resulting from annihilation of the large initial ϕ_B and $\hat{\phi}_B$ density to annihilate down to a thermal relic WIMP-like densities. For simplicity, we will consider s and \hat{s} to have the same mass m_s and the same Higgs coupling $\lambda_s = \lambda_{\hat{s}}$. In this case there are two dark matter scalars, both with the same density.

III. BARYON AND DARK MATTER DENSITIES

We first give an overview of the process. As in [15], we will consider the decay of a large baryon asymmetry, initially locked in a density of thermally decoupled heavy particles, to a baryon asymmetry in relativistic annihilons ϕ_B and $\hat{\phi}_B$ at a low temperature T_d . (For simplicity we assume the annihilons have equal mass, $m_{\phi_B} = m_{\hat{\phi}_B}$.) T_d should be less than the freeze-out temperature T_{ϕ_B} of the non-relativistic $\phi_B \hat{\phi}_B$ annihilation process due to Eq. (3), in order that B-violation due to Eq. (3) does not come into thermal equilibrium. As discussed in Appendix A, the annihilons rapidly lose energy by scattering from the thermal background, becoming non-relativistic before there is any significant change in temperature from T_d . Once non-relativistic, they will annihilate via Eq. (2) to a residual annihilon asymmetry and to equal densities of s and \hat{s} .

The gauge singlets s and \hat{s} from $\phi_B \hat{\phi}_B$ annihilation are initially relativistic. They become rapidly non-relativistic via t-channel Higgs exchange scattering with thermal background particles provided that $T_d \gtrsim 0.4$ GeV, in which case relativistic c-quarks form part of the thermal background (Appendix A). If $T_d < T_s$, where T_s is the freeze-out temperature of the s and \hat{s} scalar annihilation process from Eq. (4), the s and \hat{s} densities will annihilate down to non-thermal but thermal WIMP-like relic densities¹

Thus both the baryon asymmetry and dark matter densities will be fixed by non-relativistic annihilation processes at T_d . Since the annihilation processes from Eq. (3) and Eq. (4) are broadly of weak interaction strength when m_{ϕ_B} and m_s are in the range O(100) GeV - O(1) TeV and when λ_B and λ_s are O(0.1), which are natural assumptions for an extension of the SM, the resulting non-thermal baryon asymmetry and dark matter density will be naturally similar to each other and to a thermal relic WIMP density as long as T_d is not very small compared to the freeze-out temperatures T_{ϕ_B} and T_s [15].

A. Baryon asymmetry from $\phi_B \hat{\phi}_B$ annihilation

The non-relativistic annihilation cross-section times relative velocity for the process $\phi_B \hat{\phi}_B \rightarrow s \hat{s}$ from Eq. (3) is

$$\langle \sigma v \rangle_{\phi_B} = \frac{\lambda_B^2}{32\pi m_{\phi_B}^2} \left(1 - \frac{m_s^2}{m_{\phi_B}^2} \right)^{1/2}. \quad (5)$$

The freeze-out number density at T_d is then

$$n_{\phi_B}(T_d) \approx \frac{H(T_d)}{\langle \sigma v \rangle_{\phi_B}}. \quad (6)$$

¹ If $T_{\phi_B} > T_d > T_s$, which can occur if m_s is sufficiently light, s and \hat{s} will have purely thermal relic densities. However, as we will discuss, this requires that the s and \hat{s} masses are close to the Higgs pole.

(The $\hat{\phi}_B$ number density is the same when $m_{\phi_B} = m_{\hat{\phi}_B}$.) If, as discussed later, ϕ_B decays to baryon number $B(\phi_B)$ and $\hat{\phi}_B$ to $B(\hat{\phi}_B)$, the baryon asymmetry to dark matter ratio at present, $r_{BDM} \equiv \Omega_B/\Omega_{DM}$, is given by

$$r_{BDM} = 3(B(\phi_B) + B(\hat{\phi}_B)) \frac{m_n}{\Omega_{DM}} \frac{g(T_\gamma)}{g(T_d)^{1/2}} \left(\frac{4\pi^3}{45M_{Pl}^2} \right)^{1/2} \frac{T_\gamma^3}{\rho_c} \frac{1}{T_d} \frac{1}{\langle \sigma v \rangle_{\phi_B}}. \quad (7)$$

Here $g(T)$ is the number of relativistic degrees of freedom in equilibrium, m_n is the nucleon mass, ρ_c the critical density, T_γ is the present photon temperature and $M_{Pl} = 1.22 \times 10^{19}$ GeV. The prefactor 3 accounts for the three colours of ϕ_B . The ϕ_B mass is therefore related to the decay temperature and r_{BDM} by

$$m_{\phi_B} = 2.81 \text{ TeV} \times g(T_d)^{1/4} r_{BDM}^{1/2} (B(\phi_B) + B(\hat{\phi}_B))^{-1/2} \left(\frac{T_d}{1 \text{ GeV}} \right)^{1/2} \lambda_B \left(1 - \frac{m_s^2}{m_{\phi_B}^2} \right)^{1/4}. \quad (8)$$

B. Dark Matter Density

The annihilation cross-section times relative velocity for gauge singlet scalar dark matter [18–20] is summarized in Appendix B. If $T_d < T_s$ then the density of dark matter is a non-thermal density from $\phi_B, \hat{\phi}_B$ annihilation. The total density of s and \hat{s} dark matter is then given by

$$\Omega_{DM} = \frac{2m_s}{\rho_c} \frac{g(T_\gamma)}{g(T_d)^{1/2}} \left(\frac{4\pi^3}{45M_{Pl}^2} \right)^{1/2} \frac{T_\gamma^3}{T_d} \frac{1}{\langle \sigma v \rangle_s}. \quad (9)$$

If $T_d > T_s$ then the dark matter is purely thermal relic in nature. In this case we replace T_d by T_s in Eq. (9).

IV. RESULTS

Our aim is to understand why the baryon and dark matter densities are within an order of magnitude of each other. We therefore compute r_{BDM} as a function of the inputs $m_s, m_{\phi_B}, \lambda_s, \lambda_B$ and T_d and study how large a region of the parameter space can account for values of r_{BDM} in the range 0.1 to 10 when $\Omega_s = 0.23$. The main constraints on the model are that (i) $m_{\phi_B} > m_s$, so that $\phi_B \hat{\phi}_B$ annihilation to $s\hat{s}$ is kinematically allowed, and (ii) that $T_d < T_{\phi_B}$, so that the B-violating interaction is out of thermal equilibrium and cannot erase the asymmetry in ϕ_B and $\hat{\phi}_B$. We set the Higgs mass to $m_h = 150$ GeV and $B(\phi_B) + B(\hat{\phi}_B) = 1$ throughout.

We first consider the constraint on scalar masses when the couplings are fixed to have values $\lambda_s = \lambda_B = 0.1$. In Figure 1 we show m_{ϕ_B} for the cases $r_{BDM} = 0.1, 1.0$ and 10.0 and m_s leading to $\Omega_{DM} = 0.23$ when $T_d = 0.1 - 10$ GeV. We also show the observed baryon-to-dark matter ratio, $r_{BDM} = 0.2$. In Figure 2 we show the same for $T_d = 1 - 80$ GeV. A wide range m_{ϕ_B} is seen to be compatible with r_{BDM} being within an order of magnitude of unity, from $\mathcal{O}(1)$ TeV to a few tens of TeV for $T_d \approx 50$ GeV and from $\mathcal{O}(100)$ GeV to a few TeV for $T_d \lesssim 1$ GeV. Interestingly, the observed value of r_{BDM} favours lower values of m_{ϕ_B} , less than 2 TeV for $T_d \lesssim 80$ GeV and less than 1 TeV for $T_d \lesssim 10$ GeV, improving the prospects for production of ϕ_B and $\hat{\phi}_B$ at the LHC.

We find that in general $m_s < m_{\phi_B}$ when $\Omega_{DM} = 0.23$. There are multiple solutions for m_s with $\Omega_{DM} = 0.23$ for a given T_d . This is more clearly seen in Figure 3, which shows m_s as a function of T_d . The large m_s branch is primarily due to annihilation to WW and ZZ . In general, the freeze-out temperature is given by $T_s \approx m_s/25$, with a similar result for T_{ϕ_B} . Therefore the upper branch has $T_d < T_s$ and so the s dark matter density in this case is non-thermal. There are also two lower branches; one slightly larger than the Higgs pole at $m_s \approx 79$ GeV when $T_d \lesssim 2$ GeV, and a second at $m_s \approx 67$ GeV. For $T_d \gtrsim 3$ GeV we find that $T_s < T_d$ in this case, in which case the lower branch s density is a thermal relic density determined by annihilation to primarily b quark pairs. Both of the lower branches require that m_s is close to the Higgs pole $m_h/2$, so these solutions appear less likely than the more generic heavy m_s solution, in which case the dark matter is most likely to be non-thermal from $\phi_B \hat{\phi}_B$ annihilation.

We next consider the case where the masses are fixed to show the effect of varying λ_B and λ_s . In Figure 4 we show the case with $m_s = 120$ GeV and $m_{\phi_B} = 400$ GeV when $T_d < 10$ GeV. In Figure 5 we show the same for the case $1 \text{ GeV} < T_d < 80$ GeV. (This magnitude of m_s may be observable in the near future at direct dark matter detection experiments, while m_{ϕ_B} should be accessible to the LHC.) For these masses, values of λ_B in the range 0.004 to 0.06 will produce r_{BDM} in the range 10 to 0.1 when $T_d \approx 10$ GeV. The range of λ_B is 0.01 to 0.6 when $T_d \lesssim 1$ GeV. For $T_d \approx 50$ GeV the range of λ_B is 0.003 to 0.03. Smaller r_{BDM} favours larger λ_B . The dark matter density $\Omega_{DM} = 0.23$ requires $\lambda_s \approx 0.04$ once $T_d \gtrsim 5$ GeV. The plot of λ_s is T_d independent once $T_d \gtrsim 5$ GeV, since in this case the dark matter is produced thermally. For $T_d \lesssim 1$ GeV, $\lambda_s \gtrsim 0.1$.

In summary, if the baryon asymmetry is injected at $0.1 \text{ GeV} \lesssim T_d \lesssim 100 \text{ GeV}$ (not a very narrow range), then for masses characterized by the weak to TeV scale (a natural range for SM extensions) and couplings in the range 0.001-1 (not unusually small), r_{BDM} is within an order of magnitude of unity. The observed r_{BDM} favours larger couplings and smaller m_{ϕ_B} . For $\lambda_s \approx \lambda_B \approx 0.1$ and $T_d \lesssim 10 \text{ GeV}$ it is quite natural to have $m_{\phi_B} \lesssim 500 \text{ GeV}$ and $m_s \lesssim 200 \text{ GeV}$ when $r_{BDM} = 0.2$, in which case production of annihilons at the LHC and direct detection of s dark matter may be possible.

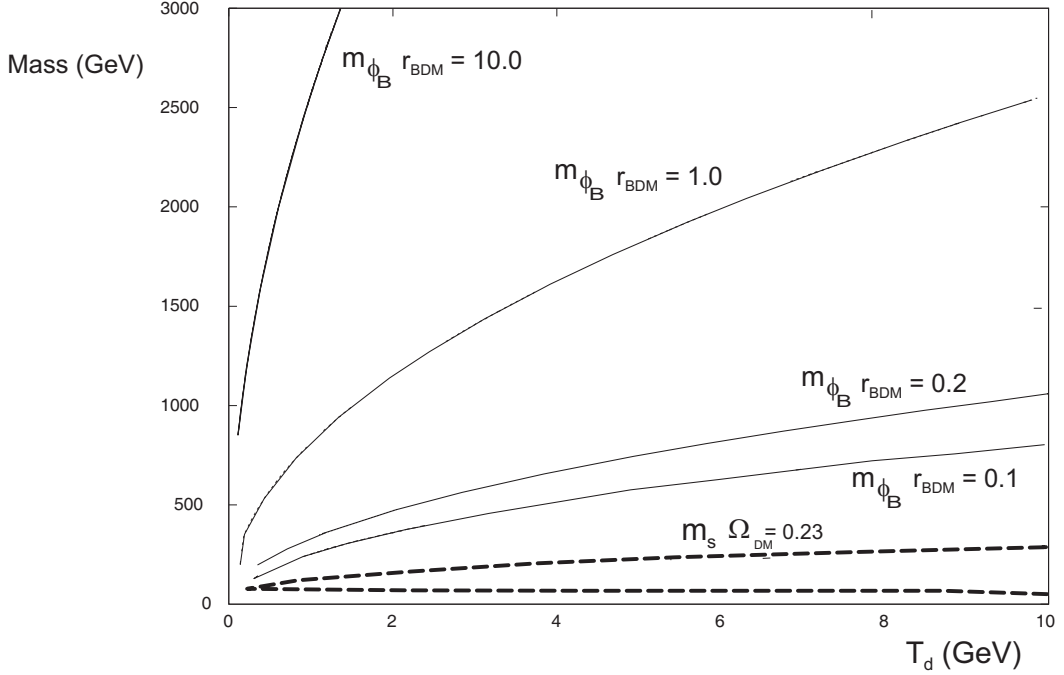


FIG. 1: Values of m_{ϕ_B} for different r_{BDM} (solid lines) and m_s for $\Omega_{DM} = 0.23$ (dashed line) as a function of T_d for the case $\lambda_B = \lambda_s = 0.1$.

V. ANNIHILON DECAY

In the previous section we showed that the baryon asymmetry and dark matter density from $\phi_B \hat{\phi}_B$ annihilation can be naturally similar to each other and to a thermal relic WIMP density. However, we still need to transfer the $\phi_B \hat{\phi}_B$ asymmetry to a conventional baryon asymmetry. At this stage the $\phi_B \hat{\phi}_B$ asymmetry does not necessarily correspond to a baryon asymmetry. This will be determined by the decay modes of ϕ_B and $\hat{\phi}_B$ to quarks. There are two possibilities: (i) baryon number is conserved by the model as a whole and ϕ_B and $\hat{\phi}_B$ have specific baryon numbers, or (ii) baryon number is conserved only by the SM sector (to ensure proton stability) and ϕ_B and $\hat{\phi}_B$ can decay to final states with different baryon numbers. In this case the effective baryon number of ϕ_B and $\hat{\phi}_B$ will be determined by their dominant decay mode to quarks. Since (i) is essentially a subset of (ii), we will concentrate on the second possibility.

The lifetimes of ϕ_B and $\hat{\phi}_B$ are necessarily long. Defining their decay temperature to SM quarks and leptons to be T_D , we require that $1 \text{ MeV} \lesssim T_D \lesssim T_d$, where the lower bound is from nucleosynthesis and the upper bound from the requirement that the initially large $\phi_B \hat{\phi}_B$ asymmetry annihilates down to a WIMP-like density at T_d prior to decaying to quarks. Therefore [15]

$$1.5 \text{ s} \gtrsim \tau \gtrsim 8 \times 10^{-11} \left(\frac{100 \text{ GeV}}{T_d} \right)^2 \text{ s}. \quad (10)$$

The long annihilon life-time requires either an extremely small renormalizable Yukawa coupling of the form $\lambda \phi_B \bar{\psi} \psi$ to SM fermions ψ ,

$$\lambda \lesssim 1.2 \times 10^{-10} \left(\frac{T_d}{1 \text{ GeV}} \right) \left(\frac{1 \text{ TeV}}{m_{\phi_B}} \right)^{1/2}, \quad (11)$$

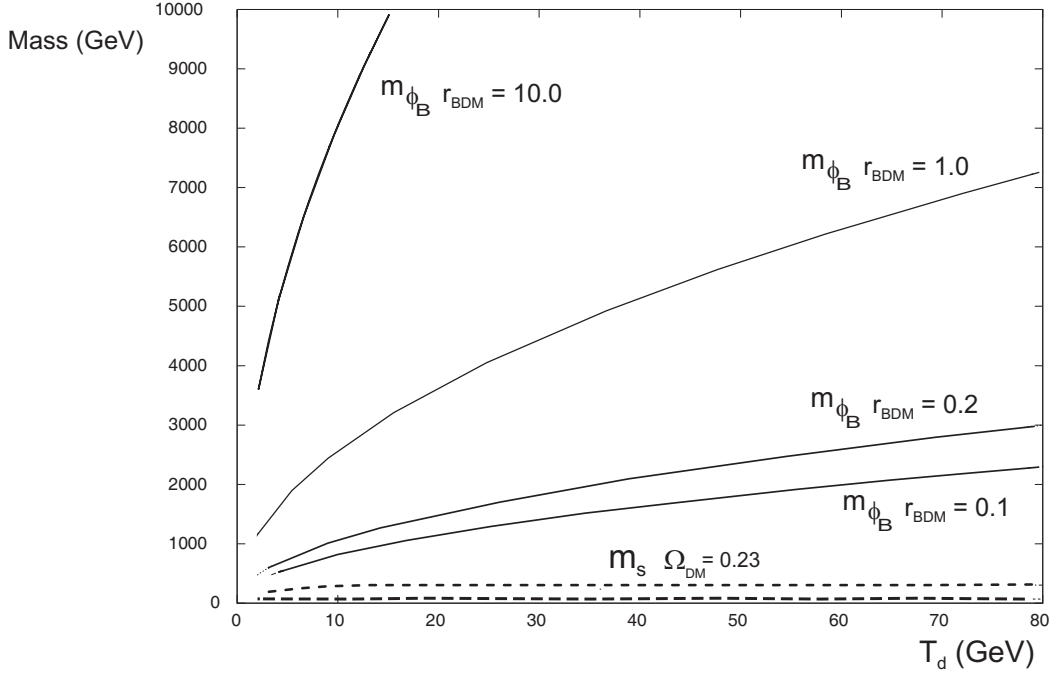


FIG. 2: Values of m_{ϕ_B} for different r_{BDM} (solid lines) and m_s for $\Omega_{DM} = 0.23$ (dashed line) as a function of T_d for the case $\lambda_B = \lambda_s = 0.1$.

or a non-renormalizable coupling suppressed by a sufficiently large mass scale. The former possibility appears explicitly unnatural, so we will consider the latter. In this case we need to explain why there are no renormalizable couplings leading to rapid ϕ_B decay.

The simplest way to achieve this is to assume that the annihilons have a large hypercharge. The largest hypercharge carried by a pair of SM fermions has magnitude $|Y| = 2$ (for $e_R^c e_R$), while the largest combinations carrying baryon number are $\bar{u}_R u_R^c$ and $\bar{d}_R d_R^c$, with $|Y| = 4/3$. The SM fermion pairs which transform as $(\mathbf{3}, \mathbf{1})$ or $(\bar{\mathbf{3}}, \mathbf{1})$ have hypercharge $|Y| = 1/3, 2/3$ or $4/3$. Therefore if $Y(\phi_B) = 5/3$ in the case where ϕ_B transforms as $(\mathbf{3}, \mathbf{1})$, there are no renormalizable couplings of ϕ_B to SM fermions². However, non-renormalizable couplings of ϕ_B and $\hat{\phi}_B$ to $d = 6$ operators are possible, for example³

$$\frac{1}{M^3} \phi_B \bar{d}_R^c d_R \bar{L}_L^c L_L \quad (12)$$

and

$$\frac{1}{M^3} \hat{\phi}_B \bar{d}_R^c e_R^c Q_L Q_L. \quad (13)$$

The mass M should then be in the range $10^6 - 10^8$ GeV to account for the low decay temperature T_D [15]. Note that for $\hat{\phi}_B$ to decay, we must assume that Z_A is slightly broken by the non-renormalizable operators. However, since these operators are suppressed by a large mass scale, this small breaking of Z_A will not introduce any dangerous mass mixing between ϕ_B and $\hat{\phi}_B$. If the operators Eq. (12) and Eq. (13) are dominant, the effective baryon number of ϕ_B and $\hat{\phi}_B$ would be $B(\phi_B) = -2/3$ and $B(\hat{\phi}_B) = -1/3$. However, if we do not assume baryon number conservation, there are other possible operators, for example

$$\frac{1}{M^3} \phi_B (\bar{e}_R Q_L \bar{e}_R L_L)^\dagger, \quad (14)$$

which allows ϕ_B to decay to a final state with $B = 1/3$. In this case the effective baryon number of ϕ_B and $\hat{\phi}_B$ will be determined by their dominant decay modes. Production of long-lived scalars with large hypercharge at the LHC, decaying to baryon number and possibly with baryon number violation in their decay modes, would therefore support this class of model.

² Note that inclusion of the Higgs doublet can only increase the dimension of an operator relative to the case without the Higgs, since the Higgs must occur in isosinglet combinations such as $H^\dagger Q$ and $H Q$, which have hypercharge equal in magnitude to SM fermions and so can be replaced by SM fermions.

³ The colour indices of the three triplets are contracted by the anti-symmetric tensor to form a $SU(3)_c$ singlet.

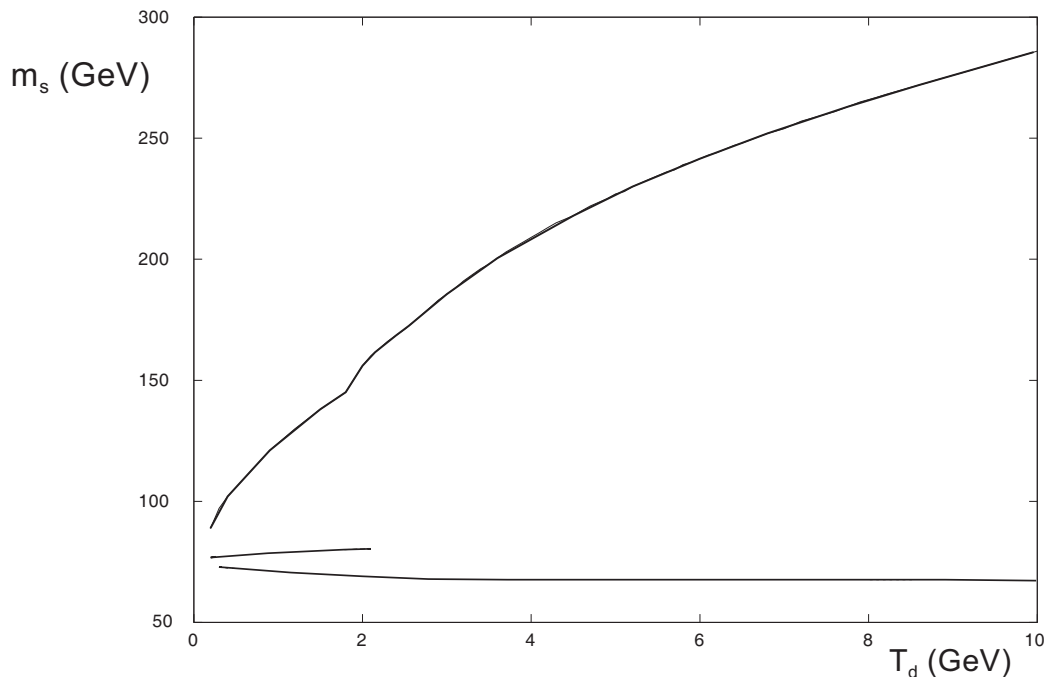


FIG. 3: Values of m_s for $\Omega_{DM} = 0.23$ as a function of T_d for the case $\lambda_B = \lambda_s = 0.1$.

VI. CONCLUSIONS

The similarity of the observed values of Ω_B and Ω_{DM} points to two distinct coincidence problems: why the density of baryons and dark matter are similar to each other and why they are both similar to a typical thermal relic WIMP density (the 'WIMP miracle'). We have shown that it is possible to explain these coincidences via a simple extension of the SM based on gauge singlet scalar dark matter and colour triplet scalar annihilons. A Z_2 discrete symmetry (Z_A) and new scalar particles are necessary to prevent dangerous B-violating interactions. The new scalars then provide a natural dark matter candidate if stabilized by a second Z_2 symmetry, Z_S . The model predicts a pair of stable scalar dark matter particles in the case where the scalars are equal in mass. The mechanism determining the final baryon asymmetry ('baryomorphosis') is based on the injection of a large baryon asymmetry in scalar annihilons at a relatively low temperature ($0.1 \text{ GeV} \lesssim T_d \lesssim 100 \text{ GeV}$), which subsequently annihilate via a B-violating interaction to a thermal relic WIMP-like density of baryons. For couplings $\lambda_B \sim 0.01 - 1$ and annihilon masses $m_\phi \sim 100 \text{ GeV} - 10 \text{ TeV}$, which are the ranges we might expect for a TeV-scale extension of the SM, the value of Ω_B/Ω_{DM} is naturally within an order of magnitude of unity. Therefore the initial large baryon asymmetry is converted to both a thermal WIMP-like baryon asymmetry and a thermal WIMP-like scalar dark matter density. Both densities are typically non-thermal, but both are determined by broadly weak strength annihilation processes and so are naturally similar to a thermal relic WIMP density. The observed baryon to dark matter ratio favours lower masses and larger couplings for the annihilons, favouring production at the LHC, and lower dark matter singlet masses, which might be observed in direct dark matter detection experiments.

The asymmetry in the annihilons is transferred to a conventional baryon asymmetry by decay to SM fermions. This must occur at a low temperature, implying a long lifetime. This suggests that renormalizable couplings of the annihilons to SM fermions must be highly suppressed or eliminated. This is most easily achieved by assigning a large hypercharge ($|Y| > 4/3$) to the annihilons. The annihilon asymmetry does not necessarily directly correspond to a baryon asymmetry. One possibility is that baryon number is conserved only in the SM sector. In this case annihilons could decay to final states with different baryon number, with the effective baryon number of the annihilons being determined by their dominant decay mode. Observation of pairs of long-lived scalars with mass $\mathcal{O}(100) \text{ GeV}$ to a few TeV, with opposite gauge charge but possibly different mass, and with large hypercharge and possibly B-violating decay modes, would therefore strongly support the class of model we have presented here.

It is important to consider whether such models are plausibly natural extensions of the SM. The model we have presented is a simple scalar particle extension of the SM with a $Z_2 \times Z_2$ discrete symmetry, which requires no fine-tuning of masses and couplings. It should be emphasized that the SM already requires an additional dark matter particle stabilized by a symmetry if dark matter is due to a WIMP, so the model might be considered an extension of this concept. The key requirements of the model are a relatively low temperature for the injection of the annihilon asymmetry and for its subsequent decay to a

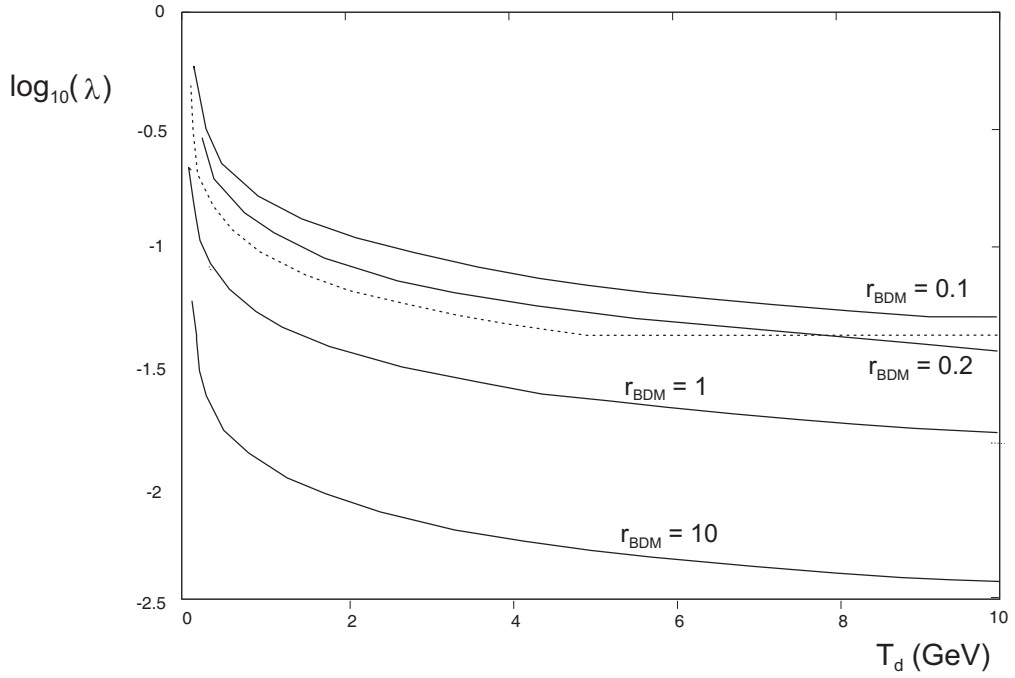


FIG. 4: Values of λ_B for different r_{BDM} (solid lines) and λ_s for $\Omega_{DM} = 0.23$ (dotted line) as a function of T_d for the case $m_{\phi_B} = 400$ GeV and $m_s = 120$ GeV.

conventional baryon asymmetry. However, a significantly wide range of injection temperature ($0.1 \text{ GeV} \lesssim T_d \lesssim 100 \text{ GeV}$) is compatible with Ω_B being within an order of magnitude of Ω_{DM} for natural scalar masses and couplings. Therefore although there is a requirement for a non-trivial sequence of processes to take place, there is nothing overtly unnatural in the requirements of the model.

The question of whether there exists a natural mechanism to relate the density of baryons and dark matter to a thermal relic WIMP density is fundamentally important to our understanding of the origin of baryons and dark matter. The model we have presented demonstrates that it is not necessary to invoke anthropic selection to explain the baryon asymmetry when dark matter is explained by the WIMP miracle. Since the new physics required is broadly at the weak or TeV scale, we can hope that experiment will be able to clarify the nature of the observed coincidence of the baryon and dark matter densities.

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Appendix A: Scattering rates from the thermal background and the slowing of relativistic scalars

In our discussion of the relic density we have assumed that particles can rapidly become non-relativistic before annihilating. Here we show that this is the case. For the case of charged annihilons, we consider the scattering rate of the annihilons from thermal background photons and show that this is rapid compared with the expansion rate and the $\phi_B \hat{\phi}_B$ annihilation rate. For the case of the relativistic gauge singlet s and \hat{s} particles produced by $\phi_B \hat{\phi}_B$ annihilation, we show that as long as relativistic c-quarks are in thermal equilibrium, scattering with SM fermions mediated by Higgs exchange can slow the singlet scalars sufficiently rapidly compared with the annihilation and expansion rate to justify treating their annihilations as non-relativistic.

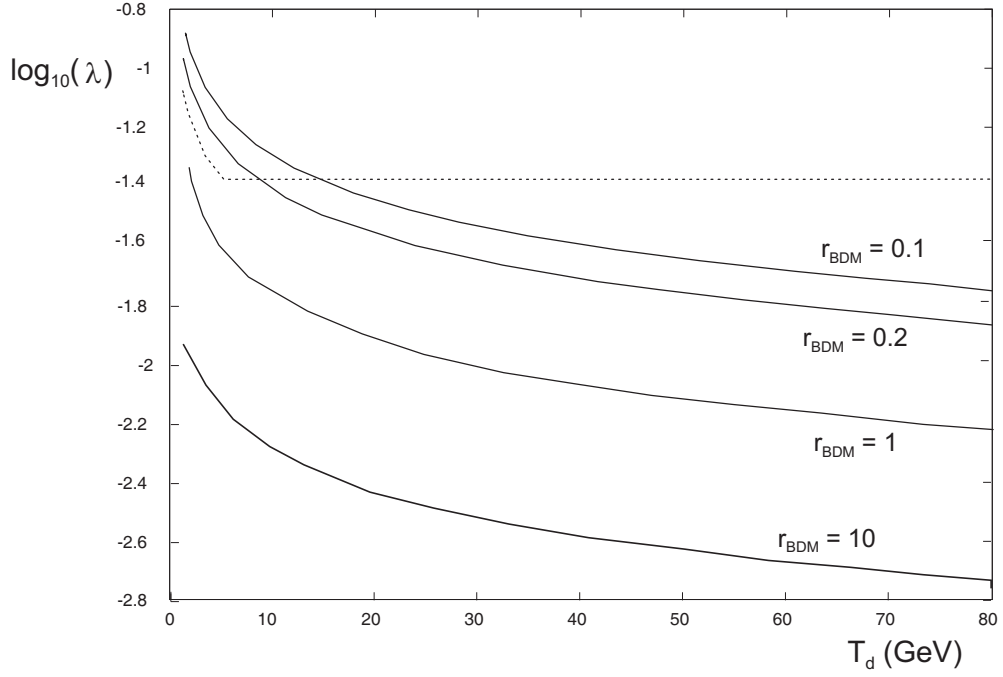


FIG. 5: Values of λ_B for different r_{BDM} (solid lines) and λ_s for $\Omega_{DM} = 0.23$ (dotted line) as a function of T_d for the case $m_{\phi_B} = 400$ GeV and $m_s = 120$ GeV.

A. Scattering and slowing of charged relativistic annihilons

For the case of charged annihilons, we consider the scattering from photons in the thermal background. The interaction term is

$$\mathcal{L}_{int} = e^2 Q^2 A_\mu A^\mu \phi_B^\dagger \phi_B, \quad (\text{A-1})$$

where Q is the electric charge of the annihilon. The scattering cross-section is then

$$\sigma = \frac{e^4 Q^4}{2\pi s}. \quad (\text{A-2})$$

In the thermal rest frame, we consider the energy of the photons on average to be $E_T \approx 3T$ and we define the energy of the ϕ_B , $\hat{\phi}_B$ to be E , where E is assumed large enough that the ϕ_B and $\hat{\phi}_B$ are relativistic. In this case $s \approx 4EE_T + m_{\phi_B}^2$. The condition for the scattering process to efficiently slow the ϕ_B particles is that

$$\frac{\Gamma_{sc}}{H} \frac{\Delta E}{E} \gtrsim 1, \quad (\text{A-3})$$

where ΔE is the energy loss per scattering and $\Gamma_{sc} = n\sigma$ is the scattering rate of the relativistic ϕ_B particles from photons, where $n \approx 2T^3/\pi^2$ is the thermal photon number density. If this is satisfied then the ϕ_B will lose most of their energy in a time shorter than H^{-1} . $\Delta E/E$ will depend on whether the ϕ_B particles are relativistic in the CM frame, which is true if $4EE_T > m_{\phi_B}^2$. In this case $s = 4EE_T$, the average energy transfer per scattering is $\Delta E = E/2$ and the condition for efficient loss of energy becomes

$$E \lesssim \frac{e^4 Q^4 M_{Pl}}{24\pi^3 k_T} \approx 1 \times 10^{13} Q^4 \text{ GeV}, \quad (\text{A-4})$$

where $k_T = (4\pi^3 g(T)/45)^{1/2}$ and we use $g(T) \approx 100$. This is easily satisfied so long as the initial energy of the ϕ_B is not very large. Therefore the ϕ_B will lose energy until they become non-relativistic in the CM frame. Once non-relativistic in the CM frame, $s = m_{\phi_B}^2$ and $\Delta E/E = 2EE_T/m_{\phi_B}^2$. The condition for efficient loss of energy then becomes

$$\frac{6e^4 Q^4 M_{Pl} E T_d^2}{\pi^3 k_T m_{\phi_B}^4} \gtrsim 1. \quad (\text{A-5})$$

This is most difficult to satisfy when $E \rightarrow m_{\phi_B}$, in which case the condition becomes

$$T_d \gtrsim 8 \times 10^{-4} \frac{1}{Q^2} \left(\frac{m_{\phi_B}}{1 \text{ TeV}} \right)^{3/2} \text{ GeV} . \quad (\text{A-6})$$

This is satisfied for $T_d \gtrsim 1 \text{ MeV}$. Therefore, for the range of T_d of interest to us here, the initially relativistic ϕ_B will become non-relativistic on a timescale short compared with H^{-1} . Since the freeze-out of the non-relativistic $\phi_B \hat{\phi}_B$ annihilation cross-section occurs once $\Gamma_{ann} \approx H$, the annihilons will become non-relativistic before they freeze-out.

B. Scattering and slowing of gauge singlet scalars

In the case of gauge singlets, the interaction with the thermal background can be much weaker, in particular at low T_d when only light fermions with small Yukawa couplings are a significant component of the thermal background.

For gauge singlets interacting with the SM via the interaction $(\lambda_s/2)s^2 H^\dagger H$, once $\langle h^o \rangle = 246 \text{ GeV}$ is introduced there is a t-channel Higgs exchange interaction with SM fermions. The average squared matrix element computed in the CM frame is

$$\overline{|\mathcal{M}|^2} = \frac{2\lambda_s^2 \lambda_f^2 \langle h^o \rangle^2 k^2 (1 - \cos \theta)}{(2k^2 (1 - \cos \theta) + m_h^2)^2} , \quad (\text{A-7})$$

where $\lambda_f = m_f / \langle h^o \rangle$ is the Yukawa coupling of SM fermion f , θ is the scattering angle in the CM frame and k is the s momentum in the CM frame, given by the solution of $2m_s k + k^2 = 4EE_T$. The cross-section is then

$$\sigma = \frac{\lambda_s^2 \lambda_f^2 \langle h^o \rangle^2 \alpha_{sc}}{64\pi s k^2} , \quad (\text{A-8})$$

where

$$\alpha_{sc} = \ln \left(1 + \frac{4k^2}{m_h^2} \right) + \left(1 + \frac{4k^2}{m_h^2} \right)^{-1} - 1 . \quad (\text{A-9})$$

If $4k^2/m_h^2 \gg 1$ then

$$\alpha_{sc} \approx \ln \left(\frac{4k^2}{m_h^2} \right) - 1 , \quad (\text{A-10})$$

while if $4k^2/m_h^2 \ll 1$ then

$$\alpha_{sc} \approx \frac{8k^4}{m_h^4} . \quad (\text{A-11})$$

We consider the limit where T_d is low compared with m_s and m_h and the s energy $E \rightarrow m_s$, which will give the least efficient transfer of energy. (We have checked that no stronger constraint results from considering $E > m_s$.) In this limit, we expect $E < m_s^2/4E_T$ and so the s will be non-relativistic in the CM frame, in which case $\Delta E/E = 2EE_T/m_s^2$ and $k = 2EE_T/m_s \ll m_h/2$. Therefore the condition for efficient loss of energy becomes

$$\frac{\lambda_s^2 \lambda_f^2 \langle h^o \rangle^2 k^3 T_d M_{Pl}}{2\pi^3 m_h^4 m_s^3 k_T} \gtrsim 1 . \quad (\text{A-12})$$

With $k = 6T_d E/m_s$ and $E = m_s$, this becomes

$$T_d \gtrsim 0.33 \text{ GeV} \times \left(\frac{5 \times 10^{-3}}{\lambda_f} \right)^{1/2} \left(\frac{0.1}{\lambda_s} \right)^{1/2} \left(\frac{m_h}{150 \text{ GeV}} \right) \left(\frac{m_s}{100 \text{ GeV}} \right)^{3/4} , \quad (\text{A-13})$$

where we have normalized λ_f to the c-quark Yukawa coupling $\lambda_c = 5 \times 10^{-3}$. c-quarks will form part of the relativistic thermal bath if $T_d \gtrsim m_c/3 = 0.4 \text{ GeV}$, therefore since in this case the bound from Eq. (A-13) is $T_d \gtrsim 0.3 \text{ GeV}$, the energy loss will be

efficient and so s particles will become non-relativistic on a timescale short compared with H^{-1} . In this case the s scalars will efficiently lose energy to the thermal background and become non-relativistic before they annihilate. However, if $T_d < 0.4$ GeV then scattering must be through s -quarks and muons. In this case $\lambda_f \approx 4 \times 10^{-4}$ and so the bound from Eq. (A-13) becomes $T_d \gtrsim 1.2$ GeV. Therefore energy loss through scattering will be ineffective and so the s particles will annihilate while relativistic. In this case the s particles become non-relativistic only via redshifting of their momentum. The final s density will therefore be fixed at the temperature T_{NR} at which they become non-relativistic rather than at T_d , resulting in an enhancement of the relic s density by a factor T_d/T_{NR} . This would require modification of our results for m_s and λ_s at very low T_d , with m_s suppressed by a factor T_{NR}/T_d for a given Ω_{DM} .

Appendix B: Gauge Singlet Scalar Annihilation Cross-Section

For convenience we provide the non-relativistic annihilation cross-section times relative velocity for gauge singlet scalars which we used in the calculation of the gauge singlet relic density. This has been discussed in [18–20]. The tree-level processes contributing to ss annihilation are (i) $ss \rightarrow hh$, (ii) $ss \rightarrow WW$, (iii) $ss \rightarrow ZZ$ and (iv) $ss \rightarrow \bar{f}f$, where f is a Standard Model fermion. (The cross-sections for $\hat{s}\hat{s}$ annihilation are similar.) (i) proceeds via a 4-point contact interaction, an s -channel Higgs exchange interaction and a t - and u -channel s exchange interaction. The resulting $\langle \sigma v_{rel} \rangle$ is

$$\begin{aligned} \langle \sigma v_{rel} \rangle_{hh} &= \frac{\lambda_s^2}{64\pi m_s^2} \left[1 + \frac{3m_h^2}{(4m_s^2 - m_h^2)} + \frac{2\lambda_s v^2}{(m_h^2 - 2m_s^2)} \right]^2 \\ &\times \left(1 - \frac{m_h^2}{m_s^2} \right)^{1/2}. \end{aligned} \quad (\text{B-1})$$

$ss \rightarrow WW, ZZ, \bar{f}f$ all proceed via s -channel Higgs exchange. The corresponding $\langle \sigma v_{rel} \rangle$ are:

$$\begin{aligned} \langle \sigma v_{rel} \rangle_{WW} &= 2 \left(1 + \frac{1}{2} \left(1 - \frac{2m_s^2}{m_W^2} \right)^2 \right) \left(1 - \frac{m_W^2}{m_s^2} \right)^{1/2} \\ &\times \frac{\lambda_s^2 m_W^4}{8\pi m_s^2 \left((4m_s^2 - m_h^2)^2 + m_h^2 \Gamma_h^2 \right)}, \end{aligned} \quad (\text{B-2})$$

$$\begin{aligned} \langle \sigma v_{rel} \rangle_{ZZ} &= 2 \left(1 + \frac{1}{2} \left(1 - \frac{2m_s^2}{m_Z^2} \right)^2 \right) \left(1 - \frac{m_Z^2}{m_s^2} \right)^{1/2} \\ &\times \frac{\lambda_s^2 m_Z^4}{16\pi m_s^2 \left((4m_s^2 - m_h^2)^2 + m_h^2 \Gamma_h^2 \right)} \end{aligned} \quad (\text{B-3})$$

and

$$\langle \sigma v_{rel} \rangle_{ff} = \frac{m_W^2}{\pi g^2} \frac{\lambda_f^2 \lambda_s^2}{\left((4m_s^2 - m_h^2)^2 + m_h^2 \Gamma_h^2 \right)} \left(1 - \frac{m_f^2}{m_s^2} \right)^{3/2}. \quad (\text{B-4})$$

Here the fermion Yukawa coupling is $\lambda_f = m_f/v$, where $v = 246$ GeV and m_f is the fermion mass (fermions should be summed over colours). Γ_h is the Higgs decay width.

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